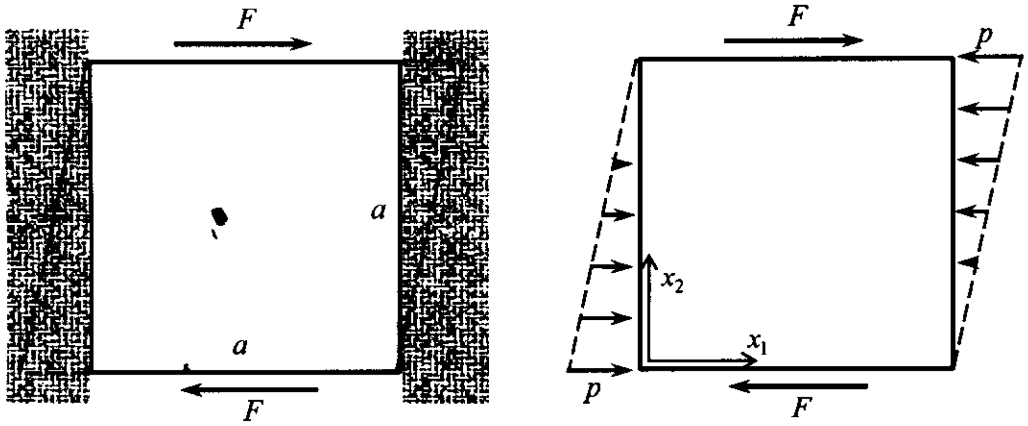
Tentamen
SOLID MECHANICS (NAVSM)
January 28 2008, 9–12 h

Question 1 A square block (in two dimensions) is held between rigid walls while being sheared at top and bottom faces by a force F (figure a). There is no adhesion nor friction between block and walls, so that the only tractions from the walls on the block can be compressive. From the expected shape change in the absence of the walls (dashed lines in figure a), we expect the linear pressure distribution indicated in figure b.



(a) (b)

In order to compute the peak value p of this distribution, the principle of virtual work is adopted with the virtual displacement field given by

$$\begin{bmatrix} \delta u_1 \\ \delta u_2 \end{bmatrix} = \begin{bmatrix} 0 & \delta\omega \\ -\delta\omega & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a. Derive the corresponding virtual strain field. What is the interpretation of this virtual displacement field?

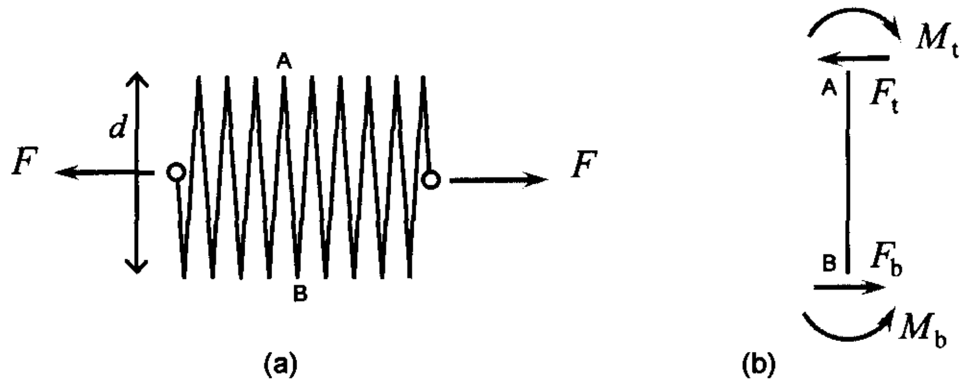
b. Determine the value of p using virtual work.

Question 2 The figure (a) below shows a two-dimensional spring, made of a zig-zag structure of slender beams of length d. When loaded at its ends by a force F, the spring extends by (predominantly) bending.

When the pitch of the spring is much smaller than d, each leaflet A–B of the spring can be considered as a vertical beam, see figure (b).

a. Express the forces and moments of this unit element at top and bottom in terms of F and d.

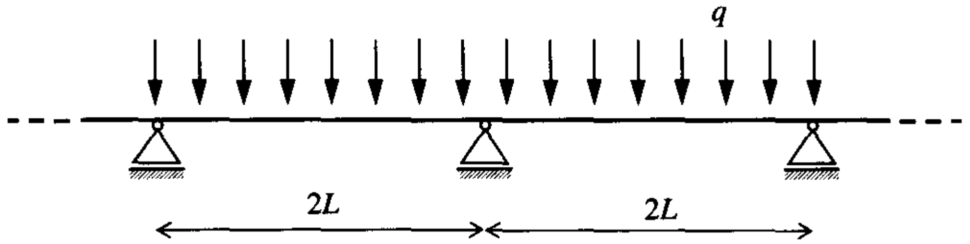
b. What are the boundary conditions to be incorporated for a beam bending analysis? (hint: make use of symmetry)



c. Compute the relative displacement of its ends A and B as a function of F, d and the bending stiffness EI of the beam.

d. Determine the spring constant for a pitch a (finite but very small: $a << d$).

Question 3 An infinitely long beam with bending stiffness EI is simply supported (i.e. can rotate freely) at points spaced $2L$ apart. The beam is subjected to a uniform distributed load q

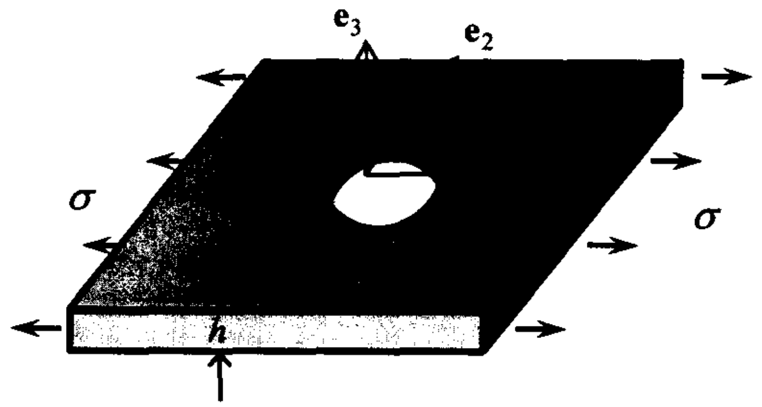


(per unit length). In order to perform a finite element computation of the deformation of this structure, it is discretized in an integer number of two-node elements. In this way, each support coincides with a node for which the displacement is henceforth going to be prescribed to be zero. For all nodes in between supports, the distributed load needs to be discretized (and even though this is not appropriate for bending beams, for the sake of this problem we will assume that each element is linear).

a. Write down the shape functions $N^I(s)$, with s a local coordinate along the element.

b. Determine the work-equivalent nodal forces for the nodes in between supports.

Question 4 A large thin plate containing a small circular hole is subjected to uniaxial tension. When it deforms elastically, stress is uniform across the thickness of the plate (i.e. in the e_3



direction), but there is a stress concentration at A (i.e. $x_1 = 0$, $x_2 = d/2$, $x_3 \in [0,h]$) of 3: $\sigma_{11} = 3\sigma$. This will change when σ becomes large enough to trigger plasticity.

The plate is made out of a polycrystalline material, so we do not know which crystal is favourably oriented and stressed for slip. Therefore, we rely on the maximum shear stress.

a. Determine the location and orientation of the plane that has the maximum (absolute value of the) shear stress (NB: consider a three-dimensional stress state, but $h << d$.)

b. How does this answer change in case a rigid cylindrical plug of diameter d was inserted in the hole prior to loading? The plug does not adhere to the plate but precludes contraction of the hole.